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Abstract. *Dynamic behavior of offshore structures is an area of extensive research, since they are widely used to support superstructures like wind turbine, offshore platforms etc. This paper, the free vibration of a continuous, elastic model of a Jack-up Platform is studied. The model is considered non-immersed and immersed in water, is under going free transverse vibration in a plane. It is modeled as a uniform Timoshenko beam (TBT) which has an tip mass on one end and is fixed at the other end. Effects of shear deformation and rotary inertia are included in the beam. Such a model is representative of numerous applications. The analytical theory for Timoshenko beam is presented, the free vibration equation is derived using Hamilton's variational principle based on Finite Element Method (FEM), which show a good agreement in results . At the end, an parametric study is carried out which provides an insight into the dependence of natural frequency on different configurations of the geometric and parameters of stiffnesses of the supports on the free vibration characteristics is investigated.*

1. Introduction

Offshore structures are large platforms that primarily provide the necessary facilities and equipment for exploration and production of oil and natural gas in a marine environment. In general, offshore structures may be used for a variety of reasons as: oil and gas exploration; production processing; accommodation; Loading and off loading facilities. There are two main categories of offshore structures, fixed and floating. Fixed structures are designed to withstand environmental forces without substantial displacement. Floating structures are designed to allow small deformations and deflections, but not negligible. [Wu and Chen 2005] solved the free vibration of non-uniform partially wet Euler-Bernoulli beam with elastic foundation and tip mass. [Wu and Chen 2010] studied the wave-induced vibrations of an axial-loaded, immersed, uniform Timoshenko beam carrying an eccentric tip mass with rotary inertia using analytical formulation. [De Rosa et al. 2013] calculated closed form solution for free vibration of a linearly tapered, partially immersed, elastically supported column with eccentric tip mass. [Ankit et al. 2016], modeled a ocean tower partially submerged, non-uniform Timoshenko beam having a rigid tip mass with eccentricity at the free end,and non-classical pile foundation at the other end.The pile foundation has been modeled as a distributed spring system which is also known as Winkler Foundation model. The damping effect in the pile-soil interaction was included by using the Kelvin-Voigt model. The monopile is widely designed as the foundation of offshore wind turbines due to its simplicity [Damgaard et al. 2014, Kuo et al. 2011].

2. Classical Theory

$$EI\left(1-\frac{P}{k'GA}\right)v^{iv}+Pv''+\rho A\ddot{v}-\rho I\left(1+\frac{E}{k'G}-\frac{P}{k'GA}\right)\ddot{v}''+\frac{\rho^2 I}{k'G}v^{iv}=0, \quad (1)$$

$$EI\left(1-\frac{P}{k'GA}\right)\psi^{iv}+P\psi''+\rho A\ddot{\psi}-\rho I\left(1+\frac{E}{k'G}-\frac{P}{k'GA}\right)\ddot{\psi}''+\frac{\rho^2 I}{k'G}\psi^{iv}=0, \quad (2)$$

in which E is the modulus of elasticity, I, the moment of inertia of cross section, k', the shear coefficient, A, the cross-sectional area, G, the modulus of rigidity, ρ the mass per unit volume, P, an initial axial tension load, v, the transverse deflection, and ψ the bending slope. Assume that the beam is excited harmonically with a frequency f and

$$v(x,t)=V(x)e^{jft}, \qquad \psi(x)=\Psi(x,t)e^{jft} \qquad \text{and} \qquad \xi=x/L, \quad (3)$$

where $j=\sqrt{-1}$, ξ is the non-dimensional length of the beam, $V(x)$ is normal function of $v(x)$, $\Psi(x)$ is normal function of $\psi(x)$, and L, the length of the beam. Substituting the above relations into Eq. (1) and Eq. (2) through Eq. (3) and omitting the common term e^{jft}, the following equations are obtained

$$(1-p^2s^2)\frac{\partial^4 V}{\partial\xi^4}+(b^2r^2+b^2s^2-p^2b^2s^2r^2+p^2)\frac{\partial^2 V}{\partial\xi^2}+(b^4r^2s^2-b^2)V=0, \quad (4)$$

$$(1-p^2s^2)\frac{\partial^4 \Psi}{\partial\xi^4}+(b^2r^2+b^2s^2-p^2b^2s^2r^2+p^2)\frac{\partial^2 \Psi}{\partial\xi^2}+(b^4r^2s^2-b^2)\Psi=0, \quad (5)$$

with

$$b^2=\frac{\rho AL^4}{EI}f^2 \qquad \text{and} \qquad f=2\pi\omega, \quad (6)$$

where f is angular frequency, and ω, the natural frequency, and

$$r^2=\frac{I}{AL^2}, \qquad s^2=\frac{EI}{k'AGL^2} \qquad \text{and} \qquad p^2=\frac{Pl^2}{EI}, \quad (7)$$

are coefficients related with the effect of rotatory inertia, shear deformation and axial load. The solutions of equations Eq.(4) and Eq.(5) may be written as [Huang 1961]:

$$V(\xi)=C_1cosh(b\alpha\xi)+C_2sinh(b\alpha\xi)+C_3cos(b\beta\xi)+C_4sin(b\beta\xi), \quad (8)$$

$$\Psi(\xi)=C_1'sinh(b\alpha\xi)+C_2'cosh(b\alpha\xi)+C_3'sin(b\beta\xi)+C_4'cos(b\beta\xi), \quad (9)$$

where the function $V(\xi)$ is know as the normal mode of the beam, C_i and C_i', with $i=1,2,3,4$, are coefficients which can be found from boundary conditions, and α and β are coefficients given as:

$$\alpha=\frac{1}{\sqrt{2}}\sqrt{-(r^2+s^2)+\sqrt{(r^2-s^2)^2+\frac{4}{b^2}}}, \quad (10)$$

$$\beta = \frac{1}{\sqrt{2}}\sqrt{(r^2+s^2)+\sqrt{(r^2-s^2)^2+\frac{4}{b^2}}}. \tag{11}$$

Note that the coefficients r and s relates the four theories of beam. These rotatory and shear dimensionless parameters relates TBT with other widely used beam theories: Euler-Bernoulli beam theory (EBT), Rayleigh beam theory (RBT) and Shear beam theory (SBT) [Jafari-Talookolaei et al. 2011]. Rayleigh and Shear beam can be formulated neglecting the shear deformation ($s = 0$) and the rotatory inertia contribution ($r = 0$) respectively. Furthermore, EBT results are obtained neglecting both effects discussed.

3. Additional Mass

The additional mass represents the fluid displaced by the movement of the cylinder. The inertia of the fluid to the system should be considered, because as the speed varies continuously the additional mass of the fluid has a permanent contribution in the dynamics of the system [Pedroso 1982].The expression for the additional mass calculation in the case of a submerged cylinder is written as:

$$M^* = \rho_{fluid}\pi r^2. \tag{12}$$

Thus the natural frequencies for the submerged condition are given by Eq.(13) in which the additional mass is included, that is:

$$\omega_s = b\sqrt{\frac{EI}{(\rho A + M^*)L^4}}. \tag{13}$$

4. Finite Element Method

The element model is showed in Fig. (1), the generalized coordinates at each node are V, the total deflection, and Ψ, the total slope. This results in a element with four degrees of freedom thus enabling the expression for V and Ψ to contain two undetermined parameters each, which can beam replaced by the four nodal coordinates.

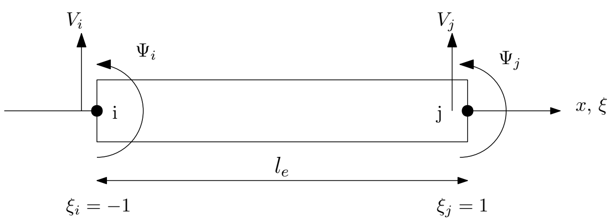

Figure 1. Beam element

Using the non-dimension coordinate (ξ) and element length l_e defined in Fig. (1) , the displacement V and total slope Ψ can be written in matrix form as follows:

$$V = [\mathbf{N}(\xi)]\,\{\mathbf{v}\}_e \qquad \text{and} \qquad \Psi = \left[\overline{\mathbf{N}}(\xi)\right]\{\mathbf{v}\}_e\,, \tag{14}$$

where

$$[\mathbf{N}(\xi)] = [N_1(\xi) \quad N_2(\xi) \quad N_3(\xi) \quad N_4(\xi)]\,, \tag{15}$$

$$[\overline{\mathbf{N}}(\xi)] = [\overline{N}_1(\xi) \quad \overline{N}_2(\xi) \quad \overline{N}_3(\xi) \quad \overline{N}_4(\xi)]. \tag{16}$$

In the current development, a cubic shape and a quadratic shape functions are proposed respectively, as follows:

$$N_i(\xi) = \sum_{i=0}^{3} \lambda_i \xi^i \qquad \text{and} \qquad \overline{N}_i(\xi) = \sum_{i=0}^{2} \overline{\lambda}_i \xi^i, \tag{17}$$

where λ_i and $\bar{\lambda}_i$ are shape functions coefficients. The displacements functions in Eqs.(15) and (16) can be expressed in terms of dimensionless parameters of rotatory and shear [Azevedo et al. 2016].

Considering a linear spring k_l, a torsional spring k_r , point mass M_c connected to beam and $a = l_e/2$, the potential and kinetic energy for element length l_e of a uniform beam are given by, respectively:

$$\mathbf{U}_e = \int_{-1}^{1} \left\{ \frac{1}{2}\frac{EI}{a}\left(\frac{\partial \Psi}{\partial \xi}\right)^2 + \frac{1}{2}\frac{EI}{as^2}\left(\frac{1}{a}\frac{\partial V}{\partial \xi} - \Psi\right)^2 + \frac{1}{2}\frac{P}{a}\left(\frac{\partial V}{\partial \xi}\right)^2 \right\} d\xi, \tag{18}$$

$$\mathbf{T}_e = \int_{-1}^{1} \left\{ \frac{1}{2}\rho A a \left(\frac{\partial V}{\partial t}\right)^2 + \frac{1}{2} r^2 \rho A a^3 \left(\frac{\partial \Psi}{\partial t}\right)^2 \right\} d\xi + \frac{1}{2} M_p \left(\frac{\partial V}{\partial t}\right). \tag{19}$$

Therefore, the element stiffness and mass matrix are respectively written by:

$$\begin{aligned} [\mathbf{k}_e] = \Bigg[& \frac{EI}{a} \int_{-1}^{1} [\overline{\mathbf{N}}(\xi)']^T [\overline{\mathbf{N}}(\xi)'] d\xi + \\ & \frac{EI}{as^2} \int_{-1}^{1} [\mathbf{N}(\xi)' - \overline{\mathbf{N}}(\xi)]^T [\mathbf{N}(\xi)' - \overline{\mathbf{N}}(\xi)] d\xi + \frac{P}{a} \int_{-1}^{1} [\mathbf{N}(\xi)']^T [\mathbf{N}(\xi)'] d\xi \Bigg], \end{aligned} \tag{20}$$

$$\begin{aligned} [\mathbf{m}_e] = \Bigg[& \rho A a \int_{-1}^{1} [\mathbf{N}(\xi)]^T [\mathbf{N}(\xi)] \, d\xi + r^2 \rho A a^3 \int_{-1}^{1} [\overline{\mathbf{N}}(\xi)]^T [\overline{\mathbf{N}}(\xi)] \, d\xi + \\ & M_c [\mathbf{N}(\xi)]^T [\mathbf{N}(\xi)] \Bigg]. \end{aligned} \tag{21}$$

5. NUMERICAL RESULTS

This section presents two numerical examples for TBT, SBT, RBT and EBT. First, five natural frequencies are calculated to a submerse and a non-submerse clamped-free beam with a tip mass ($M_c = 40 \times 10^4 \, Kg$). In order to investigate the axial load influence, natural frequencies are calculated to various percentages of critical load (η). The same geometric and material parameters values are considered for both examples. A beam of circular cross section such that $L = 100 \, m$, $k' = 0.75$, $E = 30 \times 10^9 \, Pa$, $\nu = 0.3$, $\rho = 2500 \, Kg/m^3$ are considered. Results were obtained by FEM (discretization with 30 elements). This example was adapted of [Bomtempo 2016]. Table 1 shows the first five frequencies for a submerse and non-submerse beam.

Table 1. Frequencies of a submerse and a non-submerse clamped-free beam with a tip mass.

Non-submerse beam - diameter $d = 5m$				
Frequency	TBT	SBT	RBT	EBT
ω_1	1.3191e+00	1.3195e+00	1.3206e+00	1.3209e+00
ω_2	8.4380e+00	8.4535e+00	8.5050e+00	8.5210e+00
ω_3	2.3743e+01	2.3845e+01	2.4195e+01	2.4307e+01
ω_4	4.6276e+01	4.6635e+01	4.7888e+01	4.8308e+01
ω_5	7.5477e+01	7.6366e+01	7.9568e+01	8.0711e+01
Submerse beam - diameter $d = 5m$				
Frequency	TBT	SBT	RBT	EBT
ω_{s1}	1.1148e+00	1.1152e+00	1.1161e+00	1.1164e+00
ω_{s2}	7.1314e+00	7.1445e+00	7.1880e+00	7.2016e+00
ω_{s3}	2.0066e+01	2.0153e+01	2.0449e+01	2.0543e+01
ω_{s4}	3.9110e+01	3.9414e+01	4.0473e+01	4.0828e+01
ω_{s5}	6.3790e+01	6.4541e+01	6.7247e+01	6.8213e+01

Notice that frequencies calculated for submerse (ω_s) are lower than frequencies calculated for non-submerse beams (ω) for all theories studied. Also, it is observed that difference in the frequencies become more significant with increase of the mode numbers. Furthermore, results obtained in each theory shows that the effect of shear deformation and rotatory inertia appears to be more significant in submerse medium. Table 2 shows the first four frequencies for a submerse and non-submerse beam under compressive axial load.

Table 2. Influence of axial load for frequencies of a submerse and a non-submerse clamped-free Timoshenko beam with a tip mass.

Non-submerse beam - diameter $d = 5m$				
η	ω_1	ω_2	ω_3	ω_4
0	1.3191e+00	8.4380e+00	2.3743e+01	4.6276e+01
0.4	1.0328e+00	8.1719e+00	2.3515e+01	4.6056e+01
0.8	6.0235e-01	7.8971e+00	2.3286e+01	4.5835e+01
1.0	4.9694e-02i	7.7562e+00	2.3170e+01	4.5724e+01
Submerse beam - diameter $d = 5m$				
η	ω_{s1}	ω_{s2}	ω_{s3}	ω_{s4}
0	1.1148e+00	7.1314e+00	2.0066e+01	3.9110e+01
0.4	8.7290e-01	6.9065e+00	1.9874e+01	3.8924e+01
0.8	5.0907e-01	6.6742e+00	1.9680e+01	3.8737e+01
1.0	4.1999e-02i	6.5552e+00	1.9582e+01	3.8644e+01

Observe that frequencies calculated for both cases decreases with the increase of compressive load. Also, perceive that submerse and non-submerse results present a similar behavior to observed into Table 1. Finally, for $\eta = 1$ the first frequency will become a pure imaginary value.

6. CONCLUSION

This paper presents a brief review of Timoshenko beam theory and a two-node beam element with two degrees of freedom per node based upon Hamilton's Principle. It was observed that frequencies obtained for submerse beams were lower than non-submerse beams results. This behavior was also perceived in the investigation of the influence of compressive axial force in submerse and non-submerse beam vibrations. Furthermore, was discussed that the effect of shear deformation and rotatory inertia appears to be more significant in submerse medium. Finally, it was shown on numerical examples that results obtained were in well agreement with the presented in literature.

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